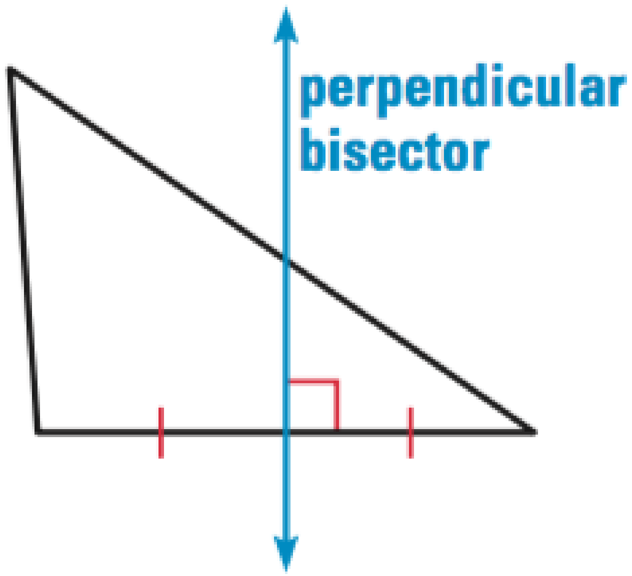
Chapter 5
Properties of Triangles

Section 2
Bisectors of a Triangle

GOAL 1: Using Perpendicular Bisectors of a Triangle

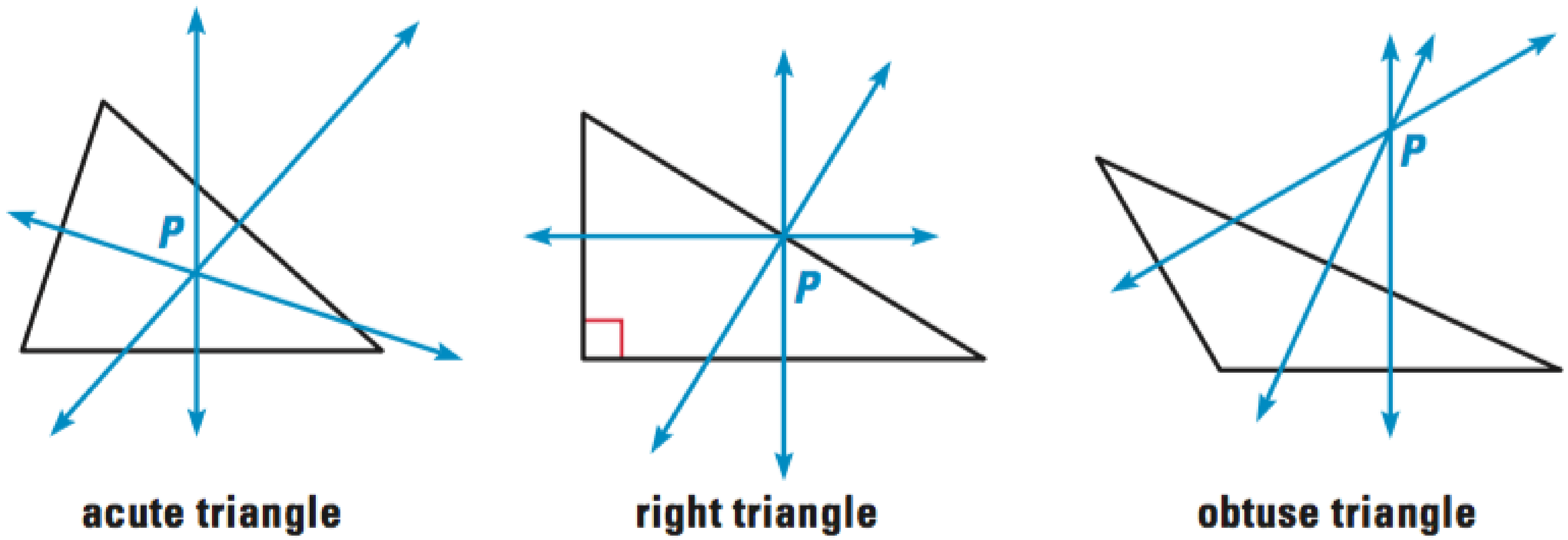
In Lesson 5.1, you studied properties of perpendicular bisectors of segments and angle bisectors. In this lesson, you will study the special cases in which the segments and angles being bisected are parts of a triangle.

A ________perpendicular bisector________ is a line (or ray or segment) that is perpendicular to a side of the triangle at the midpoint of the side.



When three or more lines (or rays or segments) intersect in the same point, they are called ___concurrent lines____ (or rays or segments). The point of intersection of the lines is called the _____point of concurrency_____.

The three perpendicular bisectors of a triangle are concurrent. The point of concurrency can be inside the triangle, on the triangle, or outside the triangle.



The point of concurrency of the perpendicular bisectors of a triangle is called the

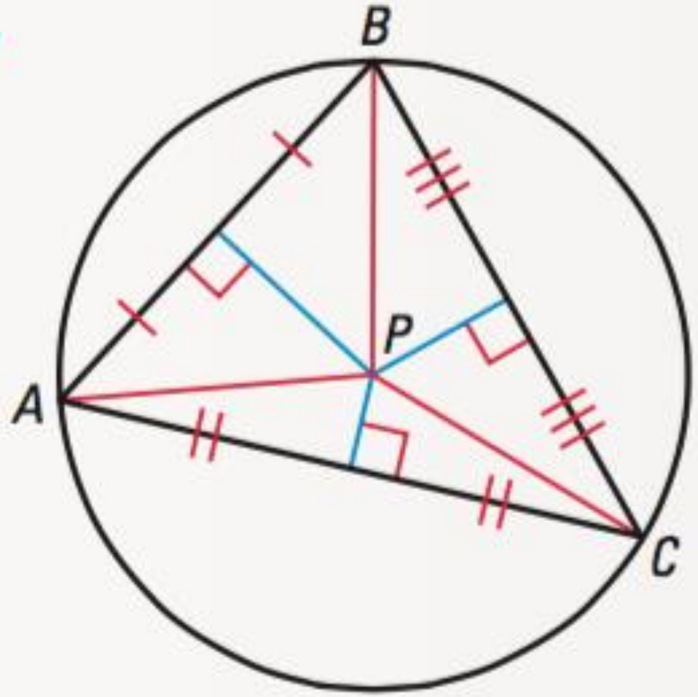
________circumcenter________. In each triangle on the previous slide, the circumcenter is at P. The circumcenter of a triangle has a special property, as described in Theorem 5.5. You will use coordinate geometry to illustrate this theorem in Exercises 29-31.

THEOREM

THEOREM 5.5 ***Concurrency of Perpendicular Bisectors of a Triangle***

The perpendicular bisectors of a triangle intersect at a point that is equidistant from the vertices of the triangle.

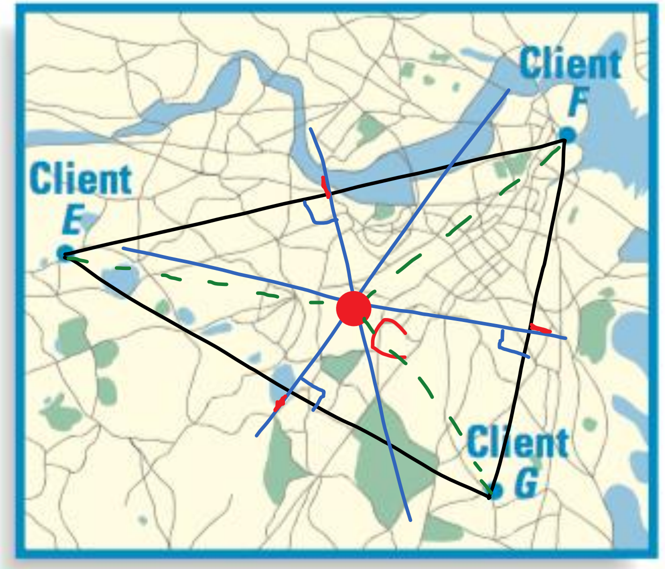
$$PA = PB = PC$$



The diagram for Theorem 5.5 shows that the circumcenter is the center of the circle that passes through the vertices of the triangle. The circle is circumscribed about ΔABC. Thus, the radius of this circle is the distance from the center to any of the vertices.

Example 1: Using Perpendicular Bisectors

Facilities Planning A company plans to build a distribution center that is convenient to three of its major clients. The planners start by roughly locating the three clients on a sketch and finding the circumcenter of the triangle formed.



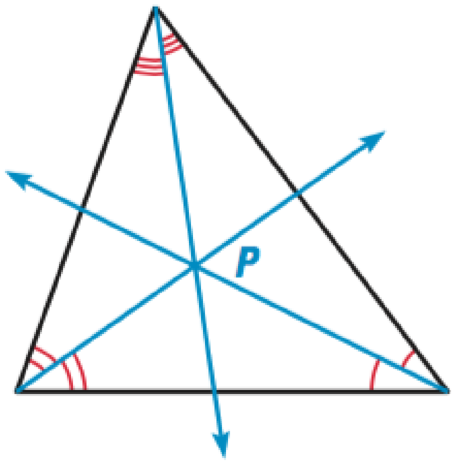
a) Explain why using the circumcenter as the location of a distribution center would be convenient for all the clients.

It would be equidistant from the 3 clients.

a) Make a sketch of the triangle formed by the clients. Locate the circumcenter of the triangle. Tell what segments are congruent. CE = CF = CG

GOAL 2: Using Angle Bisectors of a Triangle

An ______angle bisector______ is a bisector of an angle of the triangle. The three angle bisectors are concurrent. The point of concurrency of the angle bisectors is called the _____incenter_____, and it always lies inside the triangle. The incenter has a special property that is described in Theorem 5.6. Exercise 22 asks you to write a proof of this theorem.

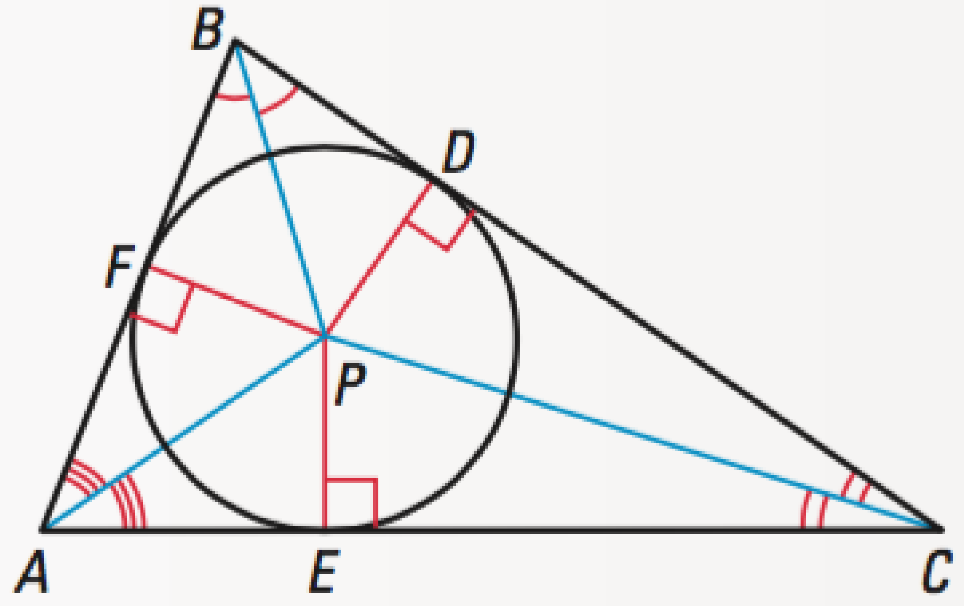


THEOREM

THEOREM 5.6 ***Concurrency of Angle Bisectors of a Triangle***

The angle bisectors of a triangle intersect at a point that is equidistant from the sides of the triangle.

$$PD = PE = PF$$



The diagram for Theorem 5.6 shows that the incenter is the center of the circle that touches each side of the triangle at once. The circle is inscribed with ΔABC. Thus, the radius of this circle is the distance from the center to any of the sides.

Example 2: Using Angle Bisectors

The angle bisectors of MNP meet at point L.

a) What segments are congruent?

LQ = LR = LS

b) Find LQ and LR.

$a^2 + b^2 = c^2$

$a^2 + 15^2 = 17^2$

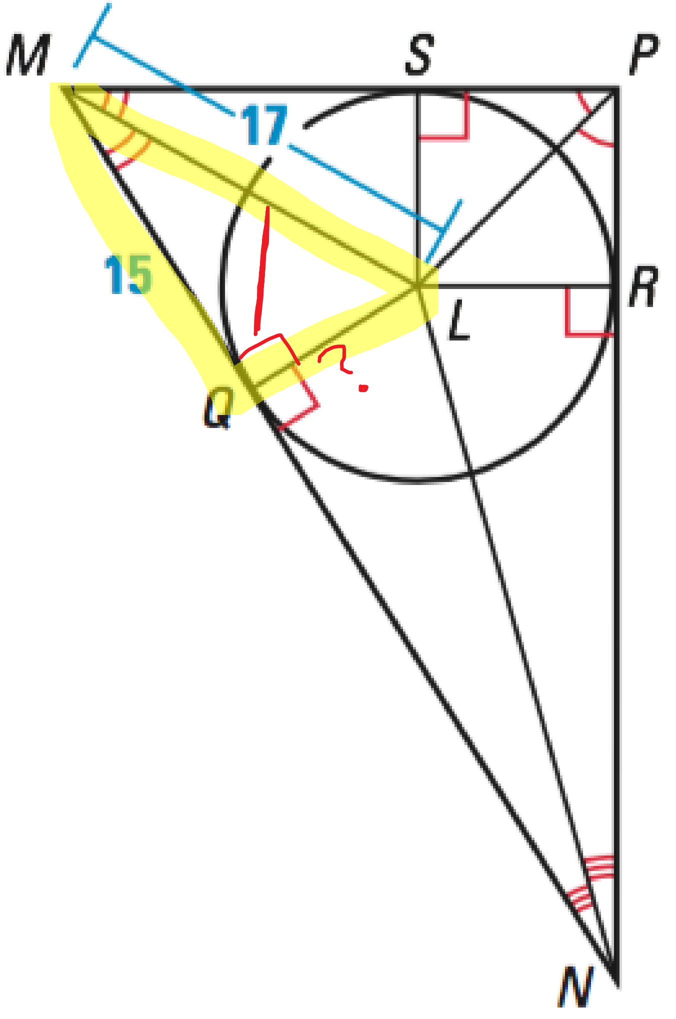
$a^2 + 225 = 289$

$\sqrt{a^{\cancel{2}}} = \sqrt{64}$

$a = 8$

$\Rightarrow$ LQ = 8

LR = 8



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